

SuperCosmology

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We present a Mathematica package for performing algebraic and numerical computations in cosmological models based on supersymmetric theories. The programs allow for (I) evaluation and study of the properties of a scalar potential in a large class of supergravity models with any number of moduli, an arbitrary superpotential, Kähler potential, and D-term; (II) numerical solution of a system of scalar and Friedmann equations for the flat FRW universe with any number of scalar moduli and arbitrary moduli space metric. We are using here a simple set of first order differential equations which we derived in a Hamiltonian framework. Using our programs we present some new results: (I) a shift-symmetric potential of the inflationary model with a mobile D3 brane in an internal space with stabilized volume; (II) a KKLT-based dark energy model with the acceleration of the universe due to the evolution of the axion partner of the volume modulus. The gzipped package can be downloaded from <http://www.stanford.edu/~prok/SuperCosmology/> or from <http://www.stanford.edu/~rkallosh/SuperCosmology/>

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I. INTRODUCTION

The studies of the cosmological aspects of supergravity and string theory have a long history, going back to the beginning of the 80's. At present, there is a new wave of interest in the cosmological aspects of string theory. The subject is rather complicated, partly due to the complexity of the analytical study of these theories. For example, to find the F-term part of the effective potential in supergravity, one should specify the expressions for the Kähler potential K and superpotential W . Even with the simplest K and W , the computation of potential is tedious, especially if there is more than one superfield, and the resulting expression is hard to analyse.

Deriving cosmological consequences from the models based on string theory and supergravity is intricate too. For non-canonical Kähler potentials (which are the rule rather than the exception) the equations of motion acquire additional velocity-dependent terms whose effects are not easily understood using intuition based on the simplest scalar field models. For the case of one field, one can always reduce the theory to the canonical form, but this method does not work for the description of the simultaneous motion of several different fields, so one should really solve the system of equations keeping the non-canonical kinetic terms throughout.

Our Mathematica-based package “SuperCosmology” is intended to simplify the study of supergravity potentials and of the cosmological models based on string theory and supergravity. The programs we describe here were used in papers [1]–[7], where only the final results of computations were presented. The purpose of this paper is to present the explanation of the programs and methods we used in computations. Also we present some new cosmological models and use them to demonstrate how our package works.

The SuperCosmology package consists of two parts. Part I has the following Mathematica nb-files:

`SuperPotential.nb`,
`SuperPotential_KKLT.nb`,
`SuperPotential_fine_tune.nb`,
`SuperPotential_D3.nb`.

In `SuperPotential.nb`, `SuperPotential_KKLT.nb`, `SuperPotential_fine_tune.nb` one finds examples from [7], [4] and [6], respectively, of using our program “SuperPotential” in computation of a scalar potential. In `SuperPotential_D3.nb`, a new example of an inflationary potential with a mobile D3 brane is studied, which is based on the D3/D7 inflationary model investigated before in [8]–[13].

In part II, we present the program “FRW” used in [1] for the numerical solution of the Friedmann equations for a system with any number of scalar fields with geometric kinetic terms of the form $\frac{1}{2}G_{ij}(\phi, \phi^*)\partial\phi^i\partial\phi^j$ specified by a metric $G_{ij}(\phi)$ on the scalar manifold. Part II has the following Mathematica nb-files:

`FRW_N2.nb`,
`FRW_DarkE.nb`,
`FRW_LateDarkE.nb`.

In `FRW_N2.nb`, we show a dark energy model based on the N=2 supergravity model [14], [15] which was also discussed in [1].

New results on the dark energy model based on the KKLT model [4], are presented in `FRW_DarkE.nb` and `FRW_LateDarkE.nb`. The examples in `FRW_DarkE.nb` describe the situation when the system has not yet reached the dS minimum (so that the scalars are still moving and Ω_D is still increasing). In `FRW_LateDarkE.nb` we study

the long term evolution including the time when the dS minimum is reached by the scalars.

II. N=1 SUPERGRAVITY POTENTIALS FIXING THE MODULI

It is believed that an N=1 d=4 supergravity may serve as an effective theory for a more fundamental string/M theory. A generic supersymmetric gravity has two types of geometries: the space-time geometry

$$ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu,$$

and the moduli space geometry

$$ds^2 = G^i_j(\phi)d\phi_i d\phi^j,$$

which both affect the cosmological models. Here we consider a class of models of N=1 supergravity with any number n_c of chiral multiplets $(\phi_i, \phi^i = (\phi_i)^*, i = 1, \dots, n_c)$, one Abelian vector multiplet and a gravitational multiplet. In application to cosmology, one primarily needs to know the total potential of the scalar fields and their kinetic terms (since these are typically non-canonical). In the general case of many vector multiplets, the kinetic function is a matrix $f_{\alpha\beta}(\phi)$ where $\alpha, \beta = 1, \dots, n_v$ with n_v the number of vector multiplets. Here we will consider a simpler picture with only one vector multiplet and one holomorphic function $f(\phi)$.

The supergravity action is defined by the functions [16]

$$W(\phi), \quad K(\phi, \phi^*), \quad \text{and} \quad f(\phi). \quad (1)$$

(We do not include constant FI terms here, however, we will have a field-dependent D-term potential.) The bosonic part of the action is (we use units in which $M_P = 1$):

$$\begin{aligned} e^{-1}\mathcal{L}_{\text{bos}} = & -\frac{1}{2}R - g_i{}^j(\hat{\partial}_\mu \phi^i)(\hat{\partial}^\mu \phi_j) - V \\ & - \frac{1}{4}(\text{Re } f)F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}i(\text{Im } f)F_{\mu\nu}\tilde{F}^{\mu\nu}. \end{aligned} \quad (2)$$

The potential consists of an F -term and a D -term:

$$V = V_F + V_D, \quad (3)$$

$$V_F = e^K [(\mathcal{D}^i W)(g^{-1})_i{}^j (\mathcal{D}_j W^*) - 3WW^*], \quad (4)$$

$$V_D = \frac{1}{2}(\text{Re } f)DD|_{\text{bos}} = \frac{1}{2}(\text{Re } f)^{-1}\mathcal{P}\mathcal{P}, \quad (5)$$

with

$$\mathcal{D}^i W = \partial^i W + (\partial^i K)W, \quad (6)$$

$$\begin{aligned} \mathcal{P}(\phi, \phi^*) &= i [\delta\phi_i \partial^i K(\phi, \phi^*)] = \\ &= i [-\delta\phi^i \partial_i K(\phi, \phi^*)], \end{aligned} \quad (7)$$

where $(\delta\phi_i, \delta\phi^i)$ defines the $U(1)$ gauge transformations of chiral superfields. The covariant derivative of ϕ_i is

$$\hat{\partial}_\mu \phi_i = \partial_\mu \phi_i - W_\mu \delta\phi_i. \quad (8)$$

Our package “SuperCosmology” is intended to help with studying models in the class presented above. In particular, it allows one to find the potential (3)-(5) for a given input (1) plus the gauge transformations of the superfields $\delta\phi_i$. We will explain now how to use part I of our package by considering the example given in `SuperPotential.nb`.

In the file `SuperPotential.nb` we use the scalar potential of the D3/D7 inflationary model with a light D7 brane moving towards a heavy stack of D3 branes in an internal space with a stabilized volume [7]. We give as input a list of complex fields ρ, S, Φ_+, Φ_- . These are components of the holomorphic field ϕ_i . (In the files, we use barred symbols for the anti-holomorphic fields, e. g. $\bar{\rho}$ is the complex conjugate field to ρ .) The real parts of the fields ρ and S correspond to the volume of the internal space and the inflaton, respectively. The next piece of input is the holomorphic superpotential $W(\phi)$, the Kähler potential $K(\phi, \phi^*)$, and the $U(1)$ gauge transformations of the scalar fields. The program calculates all important structures, including the σ -model metric $G_i{}^j(\phi, \phi^*)$ defining the kinetic terms of the chiral superfields. It also presents a matrix form of it and an expression for all covariant derivatives of the superpotential $D_i W$. This simplifies the search for the supersymmetric configurations with constant scalar fields, since we can now easily solve the equation $D_i W = 0$ in all directions ϕ_i . Next, the program calculates the F-term and the D-term potentials as functions of all fields (ϕ, ϕ^*) . We found it convenient for computations to switch to real scalar fields by splitting the complex fields into their real and imaginary parts, $\rho = \sigma + i\alpha$, so that the total potential can be given as a function of $2n_c$ real scalar fields. Finally, the program plots various sections of the potential as functions of just two fields at fixed values of other fields. The physical properties of this model are described in [7].

We supplement `SuperPotential.nb` with a few more examples. The example in `SuperPotential_KKLT.nb` shows how the stabilization of the volume was achieved in [4]. The example in `SuperPotential_fine_tune.nb` presents a detailed calculation of the fine-tuning procedure for the potential described in the Appendix F of [6].

Our fourth example presented in `SuperPotential_D3.nb`, is based on the potential for the model of inflation with D3/D7 branes [8] with volume stabilization, where the D7 brane is heavy and therefore only the D3 brane position modulus is turned on. Note that the Kähler potential in the case of a heavy stack of D3 branes and a light D7 [7], which is used in `SuperPotential.nb`, is given by

$$K = -3\ln(\rho + \bar{\rho}) - \frac{(S - \bar{S})^2}{2}. \quad (9)$$

The opposite limiting case with a light D3 and heavy D7 branes, considered in `SuperPotentialD3.nb`, is described by a Kähler potential

$$K = -3 \ln \left(\rho + \bar{\rho} - \frac{(\phi + \bar{\phi})^2}{2} \right), \quad (10)$$

in agreement with [7], [10]-[13], and the superpotential the same as in the KKLT model [4],

$$W = W_0 + Ae^{-a\rho}. \quad (11)$$

The example given in the file `SuperPotentialD3.nb` describes a situation which was not explored before and turned out to be quite attractive. We have the same “trench” in the $s = \frac{(\phi - \bar{\phi})}{2}$ direction which is to be interpreted as the inflaton direction. As shown in `SuperPotentialD3.nb`, the dependence on $\alpha = \frac{(\rho - \bar{\rho})}{2i}$ is quite simple: the potential includes $c \cos \alpha$ with c negative, and, therefore, there is a minimum at $\alpha = 0$: the same behavior as in [7]. However, the dependence on the field $\beta = \frac{(\phi + \bar{\phi})}{2}$ in this model is quite different from that in [7]. Indeed, in the model studied in [7] (see `SuperPotential.nb`), the F-term potential has a supersymmetric extremum at $\beta = 0$ which is a saddle point. This saddle point remains after lifting the potential by adding a D-term, and a stable configuration must have some $\beta \neq 0$ after β rolls down to the closest nearby minimum. In the model presented here in `SuperPotentialD3.nb`, the F-term potential also has a saddle point at $\beta = 0$. However, here after the D-term potential is added, this saddle point turns into a perfect dS minimum. Since the stabilized value of the volume modulus σ_{cr} depends on the values of α and β at the extremum, σ_{cr} here is the same as in the KKLT model [4], with $\alpha_{cr} = \beta_{cr} = 0$. This gives a good starting point for a study of stringy models of hybrid inflation (in particular, type IIB string theory compactified on $K3 \times \frac{T^2}{\mathbb{Z}_2}$ with D3/D7 branes present), based on the same mechanism of the volume stabilization as in the model of [4].

III. SOLVING FRIEDMANN EQUATIONS IN SUPERGRAVITY-BASED MODELS OF DARK ENERGY

In a series of papers [1]-[5], the issue of dark energy has been studied in the context of supergravity. In particular, in these models we have studied the future of the universe and compared the predictions of some models with observations. The more recent supernova data for $Z > 1$ in [17] were also compared with the theoretical predictions in [3], [5].

The action of a generic four-dimensional gauged supergravity which can be used for a dark energy hidden sector, includes gravity coupled to $2n$ scalar fields ϕ^i , and a potential:

$$g^{-1/2}L = -\frac{1}{2}R + \frac{1}{2}G_{ij}(\phi)\partial_\mu\phi^i\partial_\nu\phi^j g^{\mu\nu} - V(\phi). \quad (12)$$

Here $G_{ij}(\phi)$ is the metric on the moduli space, related to the Kähler potential. (In the cosmological context it is convenient to work with $2n$ real scalar fields, as in (12), rather than n complex fields).

We consider here a model with dark energy represented by the scalars ϕ^i with the Lagrangian (12). Also, we include the usual cold dark matter in the model, with the energy density ρ_M ,

$$\rho_M = \frac{C}{a^3}. \quad (13)$$

Also, we assume that the space is a flat FRW universe, $ds^2 = dt^2 - a(t)^2 d\vec{x}^2$, and that the scalar fields are homogeneous. With these assumptions, the scalar and Friedmann equations are:

$$\ddot{\phi}^i + 3\frac{\dot{a}}{a}\dot{\phi}^i + \Gamma_{jk}^i\dot{\phi}^j\dot{\phi}^k + G^{ij}\frac{\partial V}{\partial\phi^j} = 0, \quad (14)$$

$$\frac{\ddot{a}}{a} = \frac{-\rho_M + 2V - 4E_{kin}}{6}. \quad (15)$$

Here $\Gamma_{jk}^i(\phi)$ are the Christoffel symbols in the moduli space defined by the metric $G_{ij}(\phi)$, and E_{kin} is the kinetic part of the dark energy,

$$E_{kin} = \frac{1}{2}G_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j. \quad (16)$$

A. First order Friedmann equations

The Friedmann equations for homogeneous scalar fields take a particularly simple form in the Hamiltonian (first order) formulation, when the metric is a spatially flat FRW. For numerical calculations the first order equations are also much more suitable so we use them in our computations.

With the Lagrangian (12) the canonical momenta are

$$P_i = a^3(t)G_{ij}(\phi)\dot{\phi}^j, \quad (17)$$

and the Hamiltonian is

$$\mathcal{H}(P, \phi, t) = \frac{1}{2a^3(t)}G^{ij}(\phi)P_iP_j + a^3(t)V(\phi). \quad (18)$$

The equations of motion have the canonical form,

$$\dot{\phi}^i = \frac{\partial \mathcal{H}}{\partial P_i}, \quad \dot{P}_i = -\frac{\partial \mathcal{H}}{\partial \phi^i}. \quad (19)$$

Using the Hamiltonian (18) and adding the equation for the scale factor, we get a system of the first order ODEs:

$$\dot{\phi}^i = \frac{1}{a^3}G^{ij}(\phi)P_j \quad (20)$$

$$\dot{P}_i = -\frac{1}{2a^3}\frac{\partial G^{kl}}{\partial \phi^i}P_kP_l - a^3\frac{\partial V(\phi)}{\partial \phi^i} \quad (21)$$

$$\dot{a} = aH \quad (22)$$

$$\dot{H} = \frac{-\rho_M + 2V - 4E_{kin}}{6} - H^2 \quad (23)$$

Here the kinetic energy E_{kin} is given by

$$E_{kin} = \frac{1}{2a^6} G^{ij}(\phi) P_i P_j = \frac{1}{2} G_{ij}(\phi) \dot{\phi}^i \dot{\phi}^j \quad (24)$$

and ρ_M is given in (13).

In the simplest case of one dark energy scalar field with a minimal kinetic term of the form $(\dot{\phi})^2/2$ the system of equations (20), (21) reduces to a familiar form,

$$\dot{\phi} = \frac{1}{a^3} P \quad (25)$$

$$\dot{P} = -a^3 V' \quad \Rightarrow \quad \ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V' = 0 \quad (26)$$

The system (20)-(23) has to be supplemented by the initial conditions for ϕ^i , P_i , a , and H . We always set $a(t=0) = 1$, and use the first integral of motion to determine $H(t=0) = \sqrt{\frac{\rho_M + V(\phi) + (\dot{\phi})^2/2}{3}}|_{t=0}$.

The energy density ρ_D and the pressure p_D for scalar dark energy are given by

$$\rho_D = E_{kin} + V = \frac{1}{2} G_{ij}(\phi) \dot{\phi}^i \dot{\phi}^j + V(\phi), \quad (27)$$

$$p_D = E_{kin} - V = \frac{1}{2} G_{ij}(\phi) \dot{\phi}^i \dot{\phi}^j - V(\phi). \quad (28)$$

The total energy also includes the energy of matter,

$$\rho_T = \rho_M + E_{kin} + V. \quad (29)$$

The Ω parameter for dark energy (matter) is given by the ratio of the dark energy (matter) to the total energy,

$$\Omega_D = \frac{\rho_D}{\rho_T}, \quad \Omega_M = \frac{\rho_M}{\rho_T}. \quad (30)$$

Another important characteristic of the dark energy is its pressure-to-energy ratio defining the dark energy equation of state $p_D = w_D \rho_D$,

$$w_D = \frac{p_D}{\rho_D} = \frac{E_{kin} - V}{E_{kin} + V}. \quad (31)$$

The file `FRW_N2.nb` shows how we solve the Friedmann equations (20)-(23) for the $N = 2$ supergravity model [14] as discussed in the context of dark energy in [1]. Solutions of these equations with various initial conditions for the scalar fields, have an attractor behavior: the scalars eventually reach their attractor values defined by a minimum of the potential, and the universe asymptotically becomes de Sitter. Note that this takes place at $t_{\text{final}} \approx 3$. However, at this value of t_{final} , when all scalars are at their critical values, $w = -1$ and Ω_D is greater than today's observed value $\Omega_D \sim 0.72$. Nevertheless, one may still use this model to describe the dark energy of the present universe, assuming that the scalars have not reached the critical point yet (they are still evolving towards it). As we see from the plots, at $t_{\text{final}} < 0.8$ we have $\Omega_D \sim 0.72$, and at this time the equation of state function $w(t)$ takes various values, depending on the initial conditions, which are mostly different from -1 but not far from it.

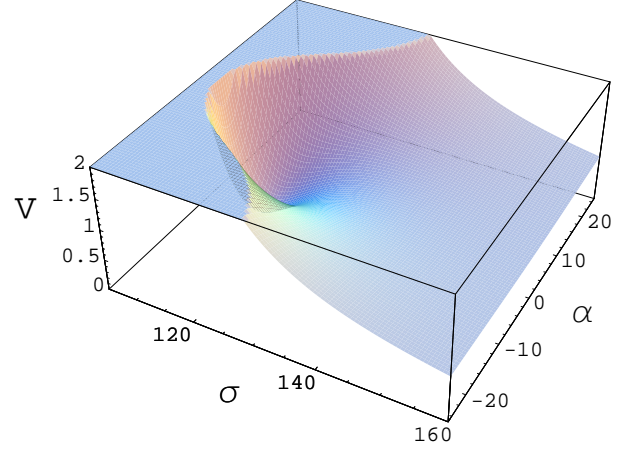


FIG. 1: Potential of the KKLT model depending on the volume σ and the axion α

B. A KKLT-based toy model of dark energy

The KKLT model [4] is given by:

$$g^{-1/2} L = -\frac{1}{2} R + \frac{3\partial\rho\partial\bar{\rho}}{(\rho + \bar{\rho})^2} - V(\rho, \bar{\rho}) \quad (32)$$

With $\rho = \sigma + i\alpha$ the kinetic term for scalars and the potential $V(\sigma, \alpha)$ are:

$$\frac{3}{4\sigma^2} [(\partial\sigma)^2 + (\partial\alpha)^2]$$

$$V = \frac{aAe^{-2a\sigma} (A(3 + a\sigma) + 3e^{a\sigma} W_0 \cos[a\alpha])}{6\sigma^2} + \frac{D}{\sigma^3}$$

We study this model in `FRW_DarkE.nb`. The model has a number of interesting features. In some cases, when one starts close to the ridge, the volume first tends to increase, however, as soon as the axion starts moving, the volume comes back and stops there. For a long time it does not change, whereas the axion tends to move quickly towards its minimum and oscillates around it. In practically all cases, Ω_D grows during the axion motion and reaches the value 0.72 at $t \approx 1$. During the same period of time $w(t)$ remains close to -1, however it deviates from this value and takes various shapes, depending on initial conditions. We also plot the acceleration parameter, $q(t) = -\frac{\ddot{a}}{a^2 H^2}$.

One has to keep in mind, that any realistic dark energy model has to accommodate today's value of the dark energy, which means that the minimum of the potential has to be close to 10^{-120} .

It is also interesting to study the late time behavior of the system. In `FRW_LateDarkE.nb`, we have solved the same equations with the same initial conditions up to the

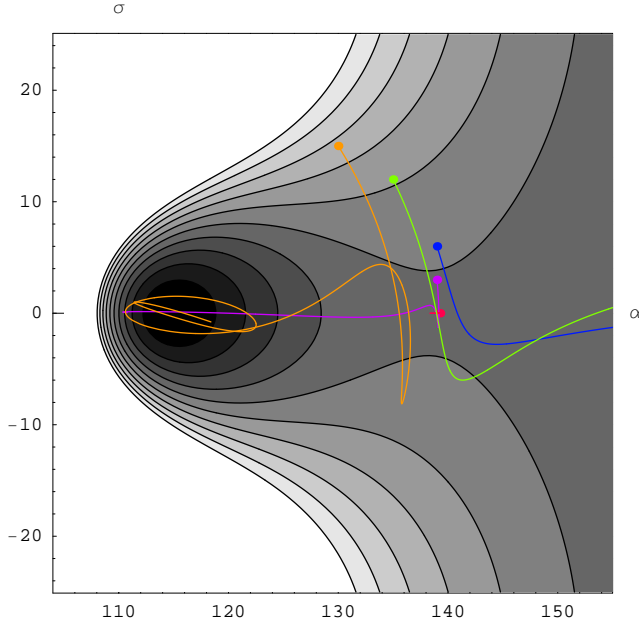
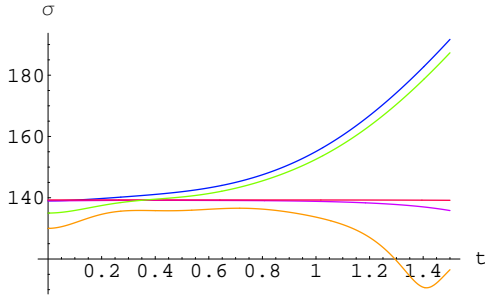
FIG. 2: Contour plot in the (σ, α) plane

FIG. 3: Volume as a function of time

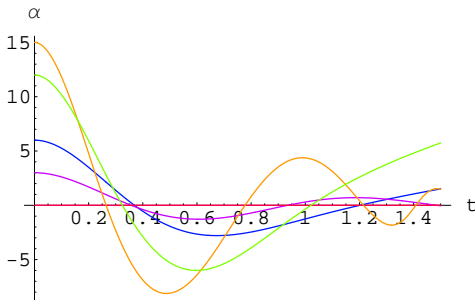
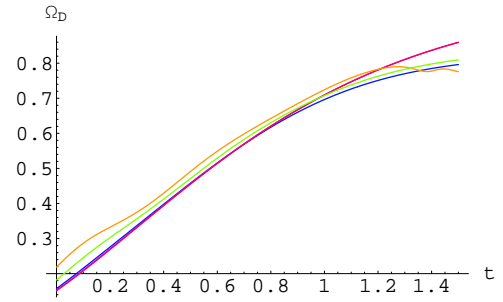
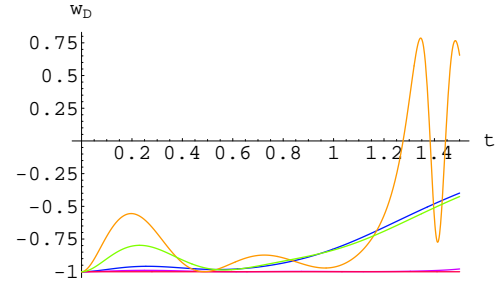


FIG. 4: Axion as a function of time

FIG. 5: Ω_D growth towards ~ 0.72 FIG. 6: Equation of state function $w(t)$

time $t \approx 30$. In most cases the system ends up at the dS minimum at $\sigma \sim 116$ and $\alpha = 0$, with $w(t) = -1$. In other cases, both the volume and the axion have a runaway behavior, and the internal 7 space de-compactifies.

IV. DISCUSSION

The SuperCosmology Mathematica package presented in this paper proved to be useful in numerous applications, both in our previous work as well as in the new models described in this paper. The interesting features of the new models are due to the special choice of potentials allowed in supersymmetric theories, and the non-canonical geometric kinetic terms.

One of the new models studied here is the KKLT-based model of dark energy. The kinetic term of the model has an $SL(2, R)$ -symmetry typical for string theory and supergravity (see [18] for earlier studies of cosmology with $SL(2, R)$ -symmetry). One can find a change of variables which will bring one of the scalars to the canonical form, $\rho = \sigma + i\alpha$, $\sigma = e^{\sqrt{2/3}\phi}$, however, the other one cannot be canonical:

$$L_{kin} = 3 \frac{\partial \rho \partial \bar{\rho}}{(\rho + \bar{\rho})^2} = \frac{1}{2} [(\partial \phi)^2 + \frac{3}{2} e^{-2\sqrt{2/3}\phi} (\partial \alpha)^2]$$

The KKLT non-perturbative potential depends on the

volume modulus σ and on the axion α and has a complicated profile. We have shown the contour plot of this potential as well as some trajectories of scalar fields in Fig. 2.

Consider, for example, the evolution of the model from the point $\sigma = 130$, $\alpha = 15$. This initial point corresponds to the top left (orange) dot on the Fig. 2. The solution obtained numerically shows that the volume modulus σ after some initial increase stops and waits, while the axion evolves towards $\alpha = 0$ and oscillates around it. Then σ moves back and eventually both fields get trapped at the minimum of the potential. The unusual behavior of the fields is explained by an interplay between the potential and non-canonical kinetic terms. From a more general perspective: in the second order equation for the scalars (14) there is an extra term $\Gamma_{jk}^i \dot{\phi}^j \dot{\phi}^k$ in addition to the standard friction due to the Hubble parameter and also the contribution of the potential depends on the metric, $G^{ij} \frac{\partial V}{\partial \phi^j}$. That makes it hard to guess, before a numerical solution of equations is found, why some trajectories end up at the minimum of the potential whereas some other trajectories lead to the volume de-compactification.

Our package “SuperCosmology” is intended to aid in the study of models related to string theory as shown in our examples. We hope it will prove to be useful for further investigations of the interface between string theory, supergravity and cosmology.

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